Extending works by Gunningham–Jordan–Vazirani and Kinnear, we present with J-V

The GL_N Skeins Partition Function for $M_\gamma = T^2 \times_\gamma S^1$

Definition

The *Skeins Partition Function* of a 3-manifold M is

$$\mathcal{Z}_M(t) := 1 + \sum_{N=1}^{\infty} \dim \mathrm{Sk}_{\mathrm{GL_N}}(M) \cdot t^N.$$

Theorem

For $\gamma \in \mathrm{SL}_2(\mathbb{Z})$, the Skeins Partition Function of M_γ admits the following expansion:

$$\mathcal{Z}_{M_\gamma}(t) = \prod_{k=1}^{\infty} (1 - t^k)^{-c_k}, \ \textit{where } c_k = \frac{1}{k} \sum_{d|k} \phi(\frac{k}{d}) |\operatorname{tr}(\gamma^d) - 2| \textit{ for generic } \gamma.$$

E.g., for $\gamma = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$, $\mathcal{Z}_{M_\gamma}(t) = 1 + 2t + 10t^2 + 36t^3 + \ldots = (1-t)^{-2}(1-t^2)^{-7}(1-t^3)^{-18} \ldots$

Want to know more? Please talk to us!

Julia Bierent
Also interested in:
Factorization Homology of wild character varieties

Matthias Vancraeynest
Also interested in:
Cluster quantisation of character varieties